APPENDIX H

Normal Distributions

In this Appendix, we illustrate how statistical problems (such as Sample Problem 6.4) can be solved using the Normal distribution table.

H.1 Standard Normal Distribution Table

Many quantities, such as Young's modulus and yield strength, are statistical in nature. Furthermore, they are typically assumed to be normally distributed with certain mean μ and standard deviation σ. The primary objective in statistical scenarios is to compute the *probability* that the quantity of interest exceeds a threshold.

As a special case, we first consider a random variable z that is normally distributed with mean 0 and standard deviation 1, i.e., $z \sim N(0,1)$, illustrated as a bell curve in Figure H.1. Given such a distribution, the *probability* that $z \leq z_0$, can be computed using the standard Normal distribution, Table H.1, as follows.

For example, the probability $P(z \leq 0.68)$ is the entry against row of 0.6 and column of 0.08 in Table H.1, i.e., $P(z \leq 0.68) = 0.75175$, as highlighted. This value is equivalent to the area under the curve to the left of $z = z_0$, as illustrated in Figure H.1.

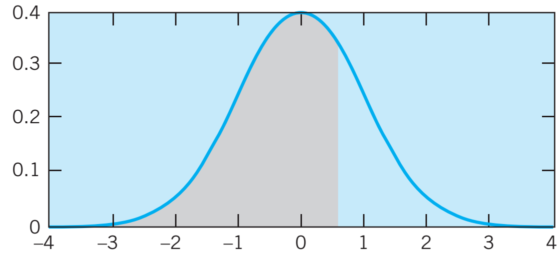

FIGURE H.1
The Normal distribution curve.

Table H.1 *The Standard Normal Distribution Table*

	Probability for a Standard Normal Variable									
	0.00	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70864	0.71226	0.71566	0.71904	0.72240
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	**0.75175**	0.75490
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94052	0.94179	0.94295	0.94408
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.80	0.95407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.95926	0.96995	0.97062
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574

(*Continued*)

Table H.1 *The Standard Normal Distribution Table (continued)*

2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
4.00	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

On the other hand, suppose we wish to compute $P(z \geq 0.68)$, then we use the fact that the total area under the Normal distribution curve is 1.0. Thus, $P(z \geq 0.68) = 1 - P(z \leq 0.68)$, i.e., $P(z \geq 0.68) = 1 - 0.75175 = 0.24825$. Finally, suppose we wish to compute $P(z \leq -0.68)$, we cannot use the table directly. However, we can use symmetry to show that $P(z \leq -0.68) = P(z \geq 0.68)$, and therefore $P(z \leq -0.68) = 0.24825$.

H.2 Converting to Standard Normal Distribution

The previous section addressed the question of computing probabilities when the underlying variable z is normally distributed with mean 0 and standard deviation of 1. Here, we consider a normally distributed variable x with mean μ and standard deviation σ, i.e., $x \sim N(\mu,\sigma)$.

In order to compute the probability $P(x \leq x_0)$, we use the following fundamental result

$$\begin{aligned} P(x \leq x_0) &= P(z \leq z_0) \\ P(x \geq x_0) &= P(z \geq z_0) \end{aligned} \quad \textbf{(H.1)}$$

where z is normally distributed with mean 0 and standard deviation of 1, and

$$z_0 = (x_0 - \mu)/\sigma \quad \textbf{(H.2)}$$

As a specific example, let us say the shear stress in a torsion bar is normally distributed with $\mu = 55\,MPa$ and $\sigma = 3\,MPa$, i.e., $\tau \sim N(55,3)\,MPa$. Further, suppose we are interested in computing the probability $P(\tau \geq 63\,MPa)$.

From the above result, $P(\tau \geq 63) = P(z \geq z_0)$ where $z_0 = (63 - 55)/2.5 = 2.67$. From the Normal distribution table, we have $P(z \geq 2.67) = 1 - P(z \leq 2.67) = 0.00379$. In conclusion, the likelihood that the shear stress will exceed $63\,MPa$ is less than 0.4%.

H.3 Linear Combination of Normal Distributions

One can take the result from the previous section one step further. Specifically, consider two normally distributed variables $x \sim N(\mu_x, \sigma_x)$ and $y \sim N(\mu_y, \sigma_y)$ that are statistically independent, i.e., x and y are not correlated. Further, let w be another variable such that $w = ax + by$, where a and b are scalar constants.

Then, one can show that w is also normally distributed with mean $a\mu_x + b\mu_y$ and standard deviation $\sqrt{(a\sigma_x)^2 + (b\sigma_y)^2}$, i.e.,

$$w \sim N(a\mu_x + b\mu_y, \sqrt{(a\sigma_x)^2 + (b\sigma_y)^2}) \quad \textbf{(H.3)}$$

For example, let $x_1 \sim N(10,0.7)$ and $x_2 \sim N(5,0.5)$ be two independent random variables. Further, let $y = 2x_1 + 2x_2$. Using the above result, the reader can verify that $y \sim N(30,1.72)$. Now, one can pose and solve probability questions such as $P(y \leq 28)$, and so on.

Let us now consider Sample Problem 6.4; the problem states that the bolts under consideration are normally distributed, having a mean twist-off strength of 20 N · m and standard deviation of 1 N · m, i.e., $x \sim N(20,1)\text{N} \cdot \text{m}$. On the other hand, the torque delivered by wrenches used in tightening these bolts are also normally distributed, having a standard deviation of 1.5 N · m, while the desired mean value must be determined, i.e., $y \sim N(\mu_y, 1.5)$.

The bolts will fail if the delivered torque exceeds their capacity. We would like to limit the probability of this event to less than 1 in 500; we enforce the condition $P(x \leq y) = 1/500 = 0.002$, i.e., $P(x - y \leq 0) = 0.002$. Let $w = x - y$. Then from Equation (H.3), $w \sim N(20 - \mu_y, 1.8028)$. Since we require $P(w \leq 0) = 0.002$, using Equations (H.1) and (H.2), we have: $P(z \leq z_0) = 0.002$, where $z_0 = (\mu_y - 20)/1.8028$. From the table this corresponds to $z_0 = -2.88$. Solving for μ_y, we have $\mu_y = 14.8 N \cdot m$.